

Multi-Objective Optimization of Impact Crusher Rotor Based on Response Surface Methodology

Li-Mei Zhao*, Lun-Jun Chen, Feng He and Yu Luo

College of Mechanical Engineering, Guizhou University, Gui yang, 550025, China

Abstract: In this study, a method of multi-objective optimization is proposed to improve the quality of crushed materials and vibration performance of the rotor. This method is driven by the first order natural frequency and the radius of the rotor. The Central Composite Design (CCD) experiment method was used to guide the selection of appropriate structure finite element analysis samples in design space. The quadratic polynomials were employed to construct response surface (RS) model based on the response outputs of these samples obtained by analyzing the first order natural frequency, the harmonic and mass with the software ANSYS. Well-distributed samples were generated in the design space by shifted Hamersley sampling method. The prominent points were selected by the weighing method as initial samples. The multi-objective genetic algorithm was used to obtain the Pareto optimal solution set. Through optimization, the first order natural frequency was increased by 5.5%; the radius of the rotor was enlarged by 2.5% and the amplitude of the vibration was decreased by 11% at the position of bearing. At the same time, the rotor mass did not change much. The results show strong engineering practicability of the proposed method.

Keywords: Finite-element analysis, multi-objective optimization, optimization design, response surface methodology.

1. INTRODUCTION

Impact crusher is a new style, high efficiency crushing equipment and is widely used in mining, metallurgy, building industry, and so on. Because of fast rotating rotor, these kinds of crushers have the problems of vibrations and loud noise in crushing operations. Structure parameters of the rotor influence not only the vibration of the machine, but also make an impact on the crusher's structure size and the crushing product quality. In recent years, optimization design of the crusher rotor has been paid more attention. One study [1] discussed the finite element modal analysis of the rotor. Natural frequency and the mode shapes were calculated through modal analysis. But the study was limited to a preliminary analysis of the vibration characteristics of a rotor body. Another study [2] analyzed stress on the rotor, and performed optimization to reduce the stress concentration by changing the rotor size. This research achieved some results, but it was only a simple optimization based on finite element analysis, limited to single design goal, and did not consider the effect of rotor structure on crushing performance in optimization process.

In this work, the domestic CF250 impact crusher was considered as the research object. A multi-objective optimization method was presented. In order to improve the vibration characteristics and the crushing product quality, the central composite design (CCD) experiment method, the response surface (RS) model, shifted Hamersley sampling method and genetic algorithm were all adopted to carry out multi-objective optimization for the rotor. This method

avoids premature phenomenon and low local searching ability of the Multi - objective evolutionary algorithm (MOEA), and provides a reference for the optimization design of other mechanical structure.

2. OPTIMIZATION MODEL

The rotor component of CF250 impact crusher is shown in Fig. (1). The parametric model was established with ANSYS code as shown in Fig. (2). Tetrahedron elements were used in the finite analysis of the rotor base and shaft. The physical parameter type of the mesh was set to mechanical.

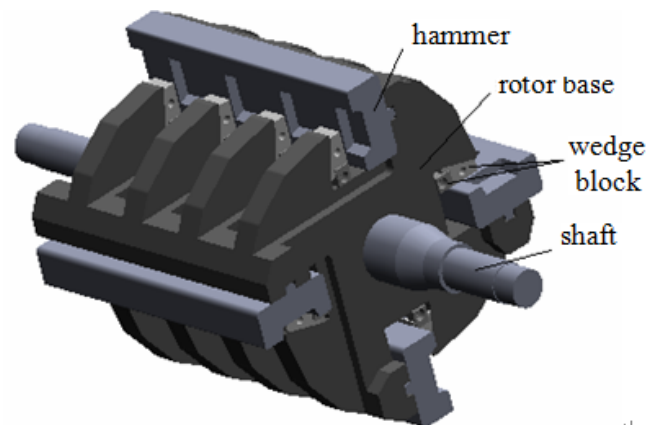


Fig. (1). Assembly drawing of rotary section.

2.1. Determination of the Objective Function

One of the optimization goals is to improve the dynamic characteristics of the rotor and to reduce the vibration of the rotor system by optimizing the structure parameters of the rotor and shaft. To improve the unit mass, stiffness of the

*Address correspondence to this author at the College of Mechanical Engineering, Guizhou University, Gui yang, 550025, China; Tel: 0851-5281078; E-mail: zlm0226@163.com

rotor can reduce the vibration of the rotor and the crusher under working condition to make the rotor vibration suitable for the intended purpose. Therefore, one of the optimization objective functions is the first natural frequency, which is closely related with the vibration of the rotor system.

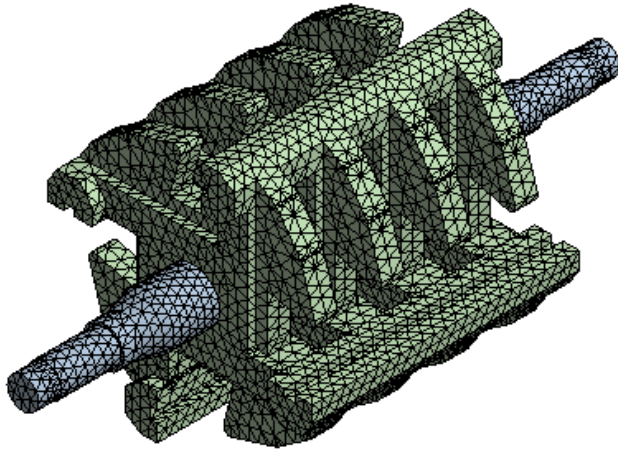


Fig. (2). Finite element model of rotor.

In order to make the hammer obtain larger kinetic energy before it collides with materials and obtain better crushing effect, the rotor radius is increased appropriately. The bigger the rotor radius, the larger the average kinetic energy per unit mass. By making the rotor radius bigger, crushing

product should be finer and the crushing product quality should be improved [3-5]. Therefore, rotor radius is considered as another optimization objective function.

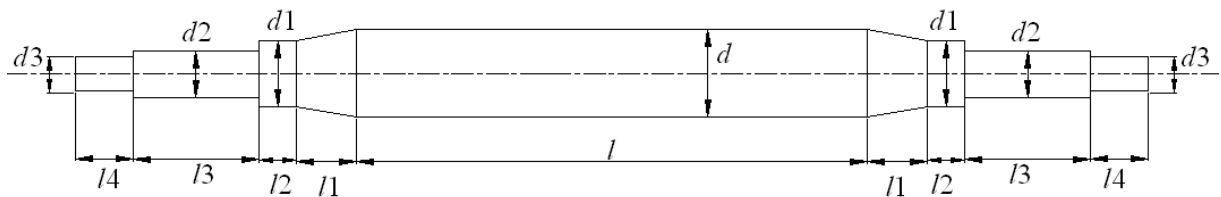
2.2. Selection of Design Variables

Generally, parameters of the rotor such as diameter, span and mass distribution may be changed to attain the goal of optimization. However, in some cases, rotor structure parameters are not simply determined by dynamics requirements of the rotor. The diameter of the rotor axis d and axis length l are restricted by performance and design requirements of the rotor and cannot be arbitrarily changed. The two rotor bearings are fixed components and their size cannot be changed. Therefore, d_2 and l_3 are restricted by rotor bearings and cannot be arbitrarily changed too. Therefore, four shaft diameters and six axis length shown in Fig. (3). were selected as design variables. Considering the fact that the rotor structure should be symmetrical, the two bilaterally symmetrical axes have a uniform size. Therefore, l_1, l_2, l_4 and d_1, d_3 were selected as design parameters. Total 7 design parameters were selected.

2.3. Multi-Objective Optimal Model

According to the above analysis, the optimization problem has two objectives; one is to increase the first-order natural frequency of the rotor to reduce the rotor vibration in the procession of work and the second is to increase the rotor radius based on the constraints fulfilled in order to increase the rotor's impact on kinetic energy and improve crushing

(a) Optimizing parameters of shaft



(b) Optimizing parameters of rotor base

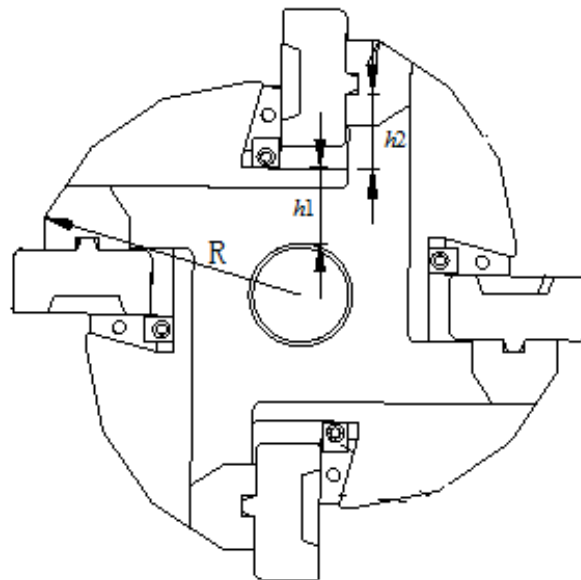


Fig. (3). Optimizing parameters of rotor.

effect. Seven dimension parameters of the rotor were evaluated as design variables. The maximum unbalance response of rotor bearing and the rotor mass are limitations in this optimization. The multi-objective optimization mathematical model of the rotor is as follows.

$$\max f_1(X)$$

$$\max R(X)$$

$$s.t. \quad \text{RESP}(X) \leq \text{RESP}_0(X)$$

$$m(X) \leq m_0$$

$$X = (l_1, l_2, l_4, d_1, d_3, h_1, h_2)^T$$

$$X \in \Omega$$

$$\underline{d}_i \leq x_i \leq \bar{d}_i \quad i = 1, 2, \dots, n$$

$$\frac{30k(\underline{\delta} + 2d)}{15k\pi - \frac{1}{2}\pi\underline{H}} \leq R \leq \frac{30k(\bar{\delta} + 2d)}{15k\pi - \frac{1}{2}\pi\bar{H}}$$

Where X is the design variable; $f_1(X)$ denotes the first-order natural frequency; $R(X)$ is the rotor radius; $m(X)$ is the rotor body mass; M_0 is the rotor body mass before optimizing; Ω is the design domain; d_i denotes the lower limit of a design variable; \bar{d}_i denotes the upper limit of a design variable; n is the number of design variables, $n=7$; $\bar{\delta}$ and $\underline{\delta}$ denote the upper and lower limit of the hammer thickness, respectively and \underline{H} and \bar{H} denote the upper and lower limit of the hammer height respectively.

3. PROCEDURES OF OPTIMIZATION DESIGN

The optimization procedures of the rotor are shown in Fig. (4). First, the mathematical model of design parameters and target parameters were established with the range of design parameters referring to the original size. The test points were selected using central composite experimental design method to obtain the response set of the test points on sample by ANSYS finite element analysis. Following this, ANSYS Workbench Design Exploration (AWB DX) optimization module was used to build the response surface model with the uniform sample in the n-dimensional feasible solution region obtained by Shifted Hamersley sampling method. Moreover, sample points were sorted through weighing function to obtain the initial population of genetic algorithm. Multi-objective genetic algorithm was employed to obtain the optimized results of the response surface to judge whether the optimized result can meet the design requirement. When the optimal solution was obtained, the result was generated, otherwise, the optimization objective function model was updated and the genetic algorithm was used to obtain optimal results.

3.1. Central Composite Experimental Design

Test points selection affects the response surface accuracy. The response surface cannot be constructed if the test point is not ideal, therefore test points should be selected by

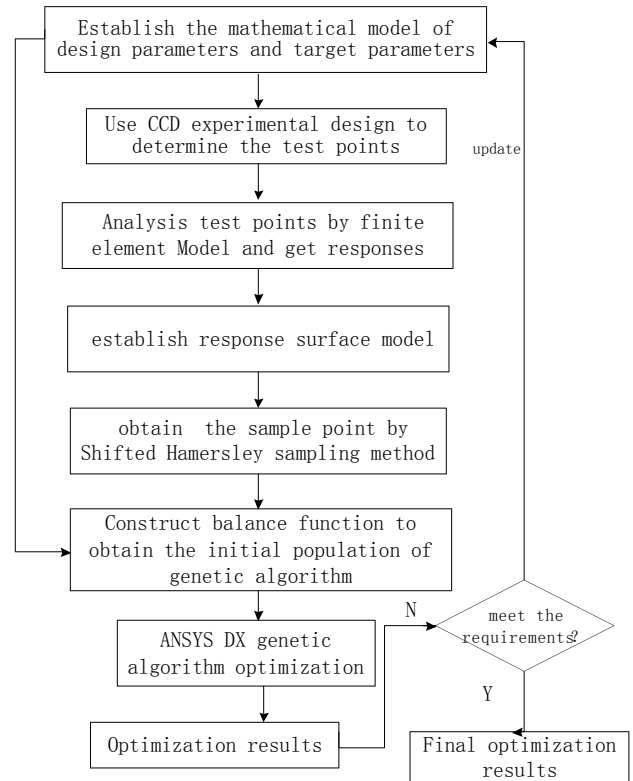


Fig. (4). Procedures of optimization design.

the experimental design theory [6-8]. Central composite design (CCD) method can provide much information and the test error by numerical experiments in the center with its extension points with minimal work cycle. When solving RS problem, the center point is evaluated equal to the structure finite element analysis result and other design points are impartially evaluated by least squares method. The central composite face (CCF) method is the most simple and quickest, in which each test variable only has three levels. CCF method is also not easy to detect error sources [9, 10]. CCF method was used to choose test points in this work.

Referring to the rotor specifications and design experience, the level and range of the design variables are shown in Table 1. Eighty nine experimental design points were constructed through CCF method.

Table 1. Value range of optimization design variables.

X(mm)						
Level	x_1	x_2	x_3	x_4	x_5	x_6
-1	140	90	300	148	30	54
0	150	100	320	152	32	60
+1	160	110	340	156	34	66

The test points were calculated By AWB DX optimization module and the response vector Y at different design points was obtained.

$$Y(X) = (f_1(X), \text{RESP-LEFT}(X), m(X))^T$$

Where $f_1(X)$ is the first natural frequency of the rotor; $RESP-LEFT(X)$ is the unbalance response of the left rotor bearing; and $m(X)$ is the rotor body mass.

3.2. Create Response Surface Model

RSM is the regression method searching the relation between several design variables and response variables. The basic function of RSM is to replace a complex model with an approximate one based on results obtained at various points. In this way, the computational burden of evaluating numerous designs is reduced. Quadratic RSM used in this work is formulated as the following polynomial function [9]:

$$y(X) = \beta_0 + \sum_{i=1}^n \beta_i x_i + \sum_{j=1}^n \beta_{ij} x_j^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^n \beta_{ij} x_i x_j \quad (1)$$

Where y is the response; x_i are design variables; β_0, β_i and β_{ij} represent unknown coefficients, and n is the number of variables. Considering that response surface model is usually used as second - order model, the response surface is expressed as:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

where $\mathbf{Y} = (y_1, y_2, \dots, y_n)^T$; $\boldsymbol{\beta} = (\beta_1, \beta_2, \dots, \beta_k)^T$; $\boldsymbol{\varepsilon}$ is an error term, $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_k)^T$;

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & \dots & x_{1k} \\ 1 & x_{21} & \dots & x_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & \dots & x_{nk} \end{bmatrix};$$

k is the number of data. The method of least squares is typically used to estimate the unknown coefficients in a multiple linear regression model in the following form:

$$\boldsymbol{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y} \quad (3)$$

Eq. (3) is substituted in equation (2), the response surface is defined by Eq. (4)

$$\mathbf{Y} \approx \mathbf{X}\boldsymbol{\beta} \quad (4)$$

When response surface is constructed, R^2 (multiple coefficient of determination) and R_a^2 (multiple adjusted coefficient of determination) are generally used to evaluate the predictive ability of the response surface [6].

$$R^2 = 1 - \frac{S_{SE}}{S_{ST}}$$

$$R_a^2 = \frac{S_{SE} / (n - k - 1)}{S_{ST} / (n - 1)}$$

$$S_{SE} = \sum_{i=1}^n (Y_i - y_i)^2$$

$$S_{ST} = \sum_{i=1}^n Y_i^2 - \left(\sum_{i=1}^n y_i \right)^2 / n$$

Response surface fitting evaluation values are shown in Table 2. It can be seen that the predictive ability of the response surface model met the requirements.

Table 2. Fitting quality.

Objective Function	Determination Coefficient R^2 (%)	Adjusted Coefficient of Determination R_a^2 (%)
The First Natural Frequency	99.32	99.15
Mass	99.22	98.26
RESP-LEFT(X)	99.17	98.82

4. MULTI OBJECTIVE OPTIMIZATION BASED ON GENETIC ALGORITHM

Normally, there are two traditional methods to solve multi-objective optimization problems. Transforming multi-objective optimization into a single objective optimization problem by weighting method or retaining only one goal and changing the other objectives into constraints. But these methods have many limitations, as the optimal solution highly depends on the designer's preference [12]. At present, genetic algorithm is often used to solve the multi-objective optimization problem. There are a large number of solving methods for the multi-objective optimization problem based on genetic algorithm. Non-dominated Sorting Genetic Algorithm- II(NSGA- II) method based on Pareto was used in this work. This method can speed up non - dominated sorting solutions and maintain the elitist and the population's diversity [13, 14]. Initial population of genetic algorithm has a great effect upon NSGA- II convergence. In order to avoid early convergence and ensure global optimum, Shifted Hamersley sampling technique and weighing function were adopted to produce initial population [15].

Hamersley sampling technique is a kind of quasi random-sampling technique based on Hamersley algorithm. Evenly distributed sample points can be produced in the n -dimensional feasible solution space through this technique [11]. In this work Shifted Hamersley sampling technique was adopted to overcome the shortcomings that Hamersley sample points showed in the region of starting point in K - dimension cube. Hamersley sample points were offset $\Delta=N/2$, and the sample points were more even, and smaller low-biased, which ensured that the multi-objective genetic algorithm can be quickly converged to the global optimal solution. In this work, 500 sample points were evenly extracted in the feasible solution region Ω using Shifted Hamersley technique.

Sorting 500 samples by weighing function as shown in Eq. (5).

$$\varphi = \sum_{i=1}^n M_i \quad (5)$$

$$M_i = \left(\frac{|y_i - y|}{y_{\max} - y_{\min}} \right)_i \quad (6)$$

Where n is the total number of objective function and constraint. M_i is defined by Eq. (6), where y_{\max} is the

Table 3. Multi-objective optimization solution set.

Ordinal	The First Natural Frequency (Hz)	Radius Related Parameters (mm)		Rotor Mass (10^3 kg)	RESP-LEFT ($10^{-2}mm$)
		h_1	h_2		
1	55.627	186.68	171.61	4.748	4.787
2	55.597	183.48	172.15	4.721	5.044
3	55.501	186.68	171.61	4.749	4.986
4	55.665	188.67	171.94	4.637	4.767
5	55.641	186.22	171.61	4.634	4.876

maximum value of $y_i(X)$, y_{\min} is $y_i(X)$ minimum, y_i is the ideal solution of objective function $y_i(X)$, y is the current value of objective function $y_i(X)$. The smaller the weight function value, the better the sample point. The first 300 Shifted Hamersley sample points were selected as the initial population of genetic algorithm.

The objective function was evaluated by genetic algorithm in AWB DX. The number of each iteration individual was 100, and the maximum operating algebra was 100 generations. The Pareto-optimal solutions obtained are as shown in Fig. (5). The abscissa and ordinate respectively represent an objective function in Fig. (5). According to the design requirements, 5 groups of solutions were selected from the Pareto optimal solution, as shown in Table 3. Considering the design goal, the rotor first order natural frequency and

the rotor radius were the most important, therefore, the fourth group of solutions was selected.

In order to modify the optimal results and make the optimization results suitable for engineering applications, the change in rotor structural response with respect to design variables was analyzed through sensitivity analysis. The sensitivity analysis results are shown in Fig. (6). According to Fig. (6), the rotor unbalance response was proportional to l_1 , l_4 , and d_1 , and d_1 were observed to be the biggest effect factors. The rotor mass unbalance response was inversely proportional to d_3 , h_1 , h_2 , and d_3 , with h_1 being the biggest effect factor. By contrast, h_2 had little effect. In addition, the first-order natural frequency of the rotor was proportional to l_1 , l_4 , d_1 , d_3 , h_1 , and l_2 , while h_1 had little effect. Therefore, it can be considered to properly increase the h_1 to reduce the

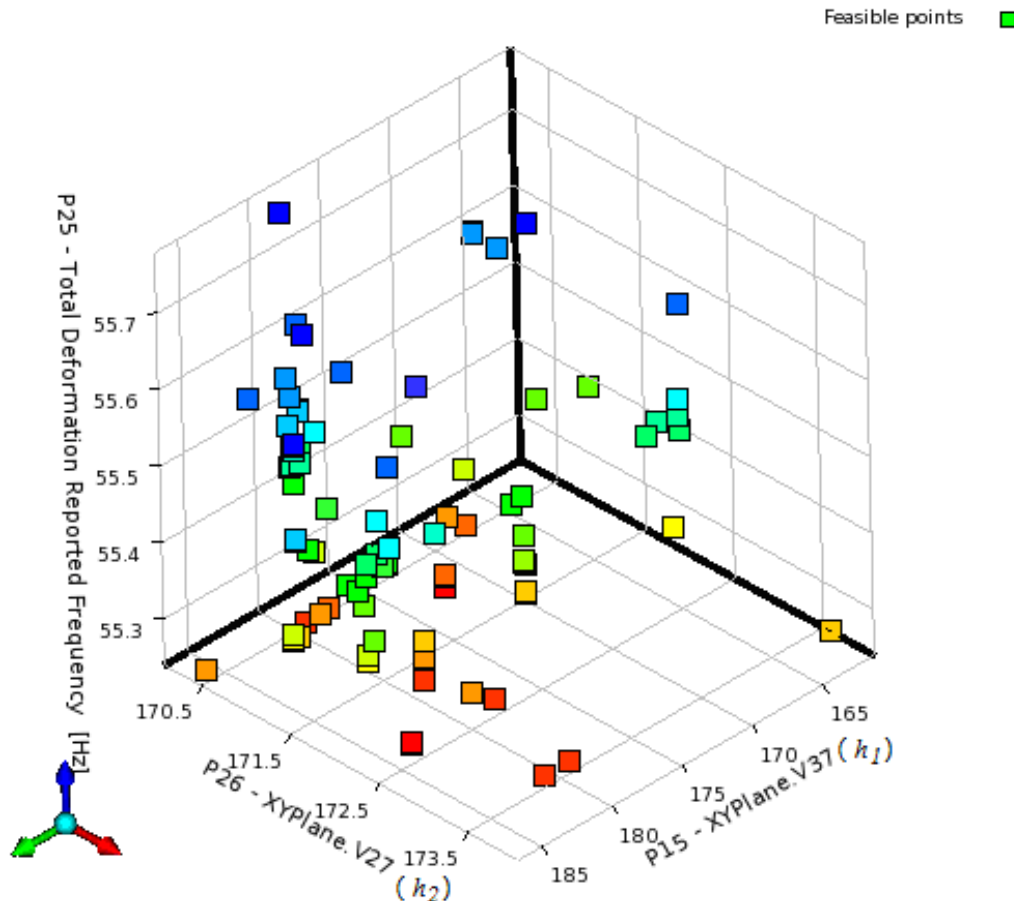


Fig. (5). Pareto optimal solution set.

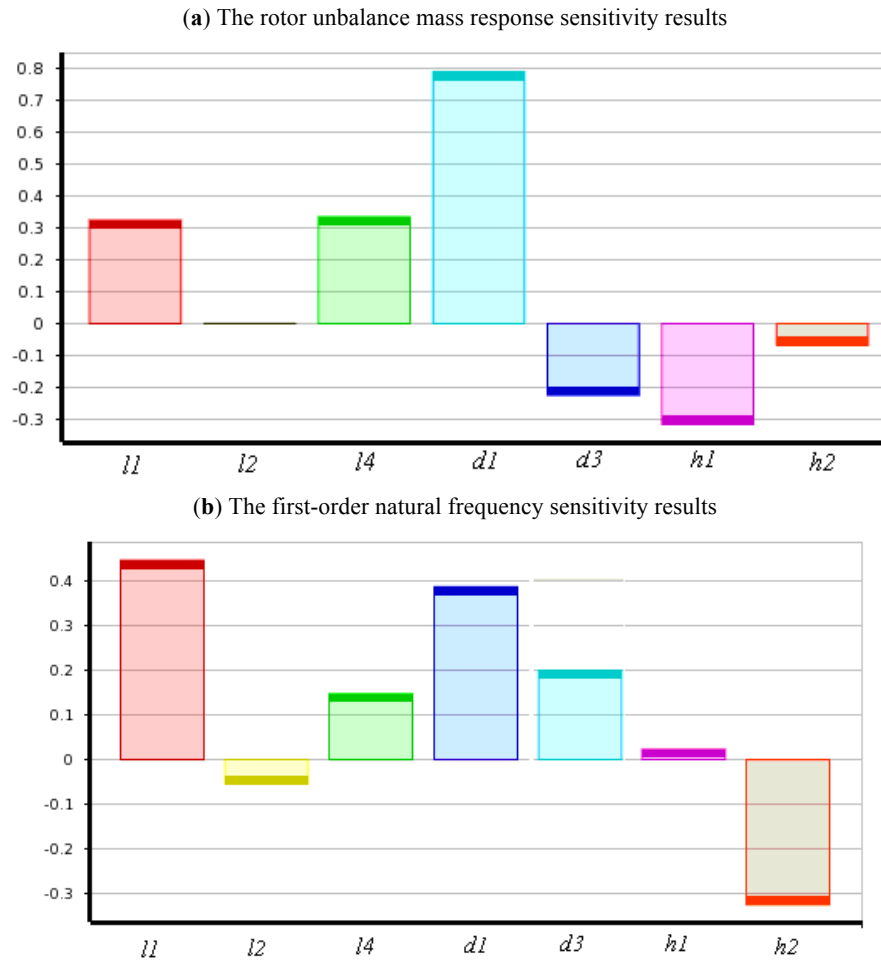


Fig. (6). Sensitivity analysis results.

rotor mass unbalance response. Increasing l_1, l_4, d_1 can improve the first-order natural frequency of the rotor, but increasing l_1, l_4 and d_1 will increase the rotor mass unbalance response, therefore the increase in l_1, l_4 and d_1 was kept limited. Increasing d_3 can enlarge the first-order natural frequency of the rotor body; at the same time, it can reduce mass unbalance response of the rotor and increase the mass of the rotor body, therefore, the increase of d_3 was kept restricted. The first-order natural frequency is inversely proportional to h_2 , and it was strongly affected by h_2 , therefore, the first order natural frequency of the rotor was increased by reducing h_2 . Optimized design variables of the

rotor were modified according to the results of sensitivity analysis as shown in Table 4.

Table 5 indicates the first-order natural frequency of the rotor which increased from 52.759HZ to 55.665HZ-a 5.5-percent. The rotor radius increased from 600mm to 615mm highlighting a 2.5-percent increase. The unbalance response of the rotor bearing decreased from 5.3777 e-2mm to 4.637e-2mm showing an 11-percent. The optimized rotor mass was 4.637 kg, which remained largely unchanged. At present, the optimization results have been applied in CF250 crusher rotor production.

Table 4. Comparison of for optimum and initial l /(mm).

Design Variables	Initial Design Variables	Optimum Design Variables	Revised Design Variables
l_1	110	119.41	120
l_4	100	95.772	96
d_1	70	71.757	72
d_3	90	94.961	95
h_1	175	188.67	190
h_2	175	171.94	170

Table 5. Comparison of performances for optimum and initial.

Optimization Variables	Parameter	Initial Performances	Optimum Performances
Objective Function	The first natural frequency $f_1(x)$ (Hz)	52.759	55.665
	Rotor radius R (mm)	600	615
State Variables	RESP-LEFT (mm)	$5.3777e^{-2}$	$4.767e^{-2}$
	Mass (10^3 kg)	4.64	4.637

CONCLUSION

- (1) The parametric finite element model of the rotor body was built as the reference model for the multi-objective optimization of the rotor body.
- (2) The quadratic response surface model, low-biased SHS and weighing function were employed to ensure quick convergence of the multi-objective genetic algorithm method. The global Pareto-optimal solution was obtained. This method is efficient and feasible and also applicable to other multi-objective optimization problems.
- (3) The method of combining the experimental design, Shifted Hamersley sampling technique, multi-objective genetic algorithm and sensitivity analysis was adopted to optimize the rotor structure. The rotor structure was analyzed to reduce vibration and improve the optimizing crushing product quality. This work proposed an improved scheme for enterprise's practical production.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflict of interest.

ACKNOWLEDGEMENTS

We wish to acknowledge the National Science and Technology Support Program of China (2011BAF07B01). We also wish to acknowledge the Science and Technology Foundation of Guizhou Province in China ([2012]2110).

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Received: September 10, 2014

Revised: November 5, 2014

Accepted: November 5, 2014

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